Stress Intensity Factors Calculation for Cracks in Bolted Joints with XFEM
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INTRODUCTION

- THE STRESS INTENSITY FACTOR IS THE MOST IMPORTANT PARAMETER FOR CRACK GROWTH ANALYSIS IN CONJUNCTION WITH THE CRACK GROWTH RATE, $da/dN$

- SIF FOR SIMPLE CONFIGURATION CAN BE OBTAINED ANALITICALLY, BUT FOR COMPLEX GEOMETRY AND LOAD STATES, NUMERICAL METHODS ARE REQUIRED

- CONTOUR INTEGRAL AND XFEM METHODS HAVE BEEN USED TO OBTAIN THE SIF IN BOLTED JOINT IN 2D AND 3D MODELS. RESULTS HAVE BEEN COMPARED WITH SIMPLIFIED 2D ANALITIC SOLUTIONS
VALIDATION OF SIMPLE CRACK GEOMETRY

- ANALYTICAL SOLUTION – 2D

\[ K = \beta_1 \cdot \sigma \cdot \sqrt{\pi \cdot c} \]

\[ \beta_1 = \frac{2W \cdot \tan \left( \frac{c}{W} \right)}{\pi \cdot c \cdot \tan \left( \frac{\pi \cdot c}{2 \cdot W} \right)} \cdot \frac{0.752 + 2.02 \cdot \left( \frac{c}{W} \right) + 0.37 \cdot \left( 1 - \frac{1}{\sin \left( \frac{\pi \cdot c}{2 \cdot W} \right) / 2} \right)^3}{\cos \left( \frac{\pi \cdot c}{2 \cdot W} \right)} \]

<table>
<thead>
<tr>
<th>c  [mm]</th>
<th>( \beta_1 ) [-]</th>
<th>SIF [MPa*mm**0.5]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.36666</td>
<td>766.0</td>
</tr>
</tbody>
</table>

\( \sigma = 100 \text{ MPa} \)

\( t = 2 \text{ mm} \)
VALIDATION OF SIMPLE CRACK GEOMETRY

- ABAQUS 2D CONTOUR INTEGRAL
  - Mesh size around crack tip 1 mm and global element size 5 mm.
  - Element type selected for the analysis is CPS8R (Plane strain elements give same result – SIF is independent of E for this type of analysis)

<table>
<thead>
<tr>
<th>Contour</th>
<th>KI [MPa*mm**0.5]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>766.9</td>
</tr>
<tr>
<td>2</td>
<td>766.5</td>
</tr>
<tr>
<td>3</td>
<td>766.7</td>
</tr>
<tr>
<td>4</td>
<td>766.7</td>
</tr>
<tr>
<td>5</td>
<td>766.7</td>
</tr>
</tbody>
</table>

DIFFERENCE 0.09%
VALIDATION OF SIMPLE CRACK GEOMETRY

- **ABAQUS 3D CONTOUR INTEGRAL**
  - 2 elements in the through thickness direction.
  - Element type C3D20R

<table>
<thead>
<tr>
<th>Position [mm]</th>
<th>Contour 1</th>
<th>Contour 2</th>
<th>Contour 3</th>
<th>Contour 4</th>
<th>Contour 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>760.1</td>
<td>757.7</td>
<td>758.3</td>
<td>758.2</td>
<td>758.3</td>
</tr>
<tr>
<td>0.5</td>
<td>818</td>
<td>818.1</td>
<td>818.1</td>
<td>818.1</td>
<td>818.8</td>
</tr>
<tr>
<td>1</td>
<td>834</td>
<td>833.3</td>
<td>834.1</td>
<td>834</td>
<td>834.1</td>
</tr>
<tr>
<td>1.5</td>
<td>818</td>
<td>818.1</td>
<td>818.1</td>
<td>818.1</td>
<td>818.8</td>
</tr>
<tr>
<td>2</td>
<td>760.1</td>
<td>757.7</td>
<td>758.3</td>
<td>758.2</td>
<td>758.3</td>
</tr>
</tbody>
</table>

Out of plane constraint effect due to thickness
More elements required
ABAQUS 3D CONTOUR INTEGRAL – SIF variation along thickness

- Through thickness mesh refinement: 5, 8 elements
- Surface mesh refinement

SIF 3D_FEM ≈1.07 SIF 2D
ABaqus 3D Contour Integral – J-integral variation along thickness

- $t = 2\,\text{mm},\, 50\,\text{mm},\, 100\,\text{mm}$
VALIDATION OF SIMPLE CRACK GEOMETRY

- ABAQUS 3D XFEM
  - Mesh size around crack tip 0.3 mm and global element size is 3 mm.
  - 4 elements along thickness direction
  - Element type C3D8R.

![Diagram of crack geometry and XFEM SIF](image-url)
VALIDATION OF SIMPLE CRACK GEOMETRY

- ABAQUS 3D XFEM– MESH SENSITIVITY
## RESULTS COMPARISON

### 2D MODEL

<table>
<thead>
<tr>
<th></th>
<th>Average SIF [MPa*mm**0.5]</th>
<th>Relative error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical Solution</td>
<td>766.0</td>
<td>--</td>
</tr>
<tr>
<td>Contour Integral 2D</td>
<td>766.7</td>
<td>0.09%</td>
</tr>
</tbody>
</table>

### 3D MODEL

<table>
<thead>
<tr>
<th></th>
<th>Average SIF [MPa*mm**0.5]</th>
<th>Relative error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contour Integral 3D refined model - C3D20R</td>
<td>825.7</td>
<td>--</td>
</tr>
<tr>
<td>XFEM</td>
<td>846.0</td>
<td>2.45%</td>
</tr>
<tr>
<td>XFEM refined</td>
<td>844.1</td>
<td>2.23%</td>
</tr>
</tbody>
</table>
SIF ANALYSIS IN BOLTED JOINT

- **BOLTED JOINT 3D CONFIGURATION**

- **BOLTED JOINT 3D – FE MODEL**
**CONTOUR INTEGRAL 2D RESULTS**

- $\sigma_1 = 100 \, \text{MPa}$
- $t = 1 \, \text{mm}$
- $R = 2.4 \, \text{mm}$
- $c = 4.0 \, \text{mm}$
- $W = 19.2 \, \text{mm}$
- $P = 1920 \, \text{N}$
- $c = 4.0 \, \text{mm}$
- $H = 99.2 \, \text{mm}$
- $R = 2.4 \, \text{mm}$

<table>
<thead>
<tr>
<th>Contour</th>
<th>$K_I$ [MPa$\cdot$mm$^{0.5}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>536.4</td>
</tr>
<tr>
<td>2</td>
<td>536.1</td>
</tr>
<tr>
<td>3</td>
<td>536.2</td>
</tr>
<tr>
<td>4</td>
<td>536.2</td>
</tr>
<tr>
<td>5</td>
<td>536.2</td>
</tr>
</tbody>
</table>
SIF ANALYSIS IN BOLTED JOINT

- CONTOUR INTEGRAL 3D RESULTS

![Contour Integral Graph]

- Position through thickness [mm]
- SIF [MPa*mm²*0.5]

- CI_Average
SIF ANALYSIS IN BOLTED JOINT

- XFEM 3D RESULTS

![](image1.png)

![XFEM average graph showing stress distribution through thickness](image2.png)
SIF ANALYSIS IN BOLTED JOINT

- SIF RESULTS COMPARISON

![Graph showing SIF comparison](image)

- Difference +7%
- Difference -18%
Thank you for your attention

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